International Journal of Theoretical Physics, Vol. 43, No. 1, January 2004 (© 2004)

Massive Photons and the Volkov Solution

Miroslav Pardy[1,2]

Received September 1, 2003

This article applies the theory of massive electrodynamics to the Dirac equation with the aim to find the generalized Volkov solution with massive photon field. The resulting equation is the Riccati equation which cannot be solved in general. We use the approximative Volkov function for massive photons and consider an electron in the periodic field and in the laser pulse of the δ-function form. We derive the modified Compton formulas for the interaction of the multiphoton object with an electron for both cases.

KEY WORDS: Volkov solution; Riccati equation; massive photons; Compton effect.

1. INTRODUCTION

The introduction of the massive photon into field theory is elementary from the mathematical point of view. However, the physical reasons for such generalization require serious motivation.

We know from the special theory of relativity, that the relativistic mass formula

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1}$$

where m_0 is the rest mass, has physical meaning for $v = c$, only if $m_0 = 0$. Since the velocity of photon in vacuum is $v = c$, it follows from the viewpoint of the special theory of relativity that the rest mass of photon is zero.

Nevertheless, massless photon has a momentum

$$p = \frac{E}{c} = \frac{\hbar\omega}{c}, \tag{2}$$

as it follows from the Einstein relativistic mass formula $E = \sqrt{c^2 p^2 + m^2 c^4}$ in which we put the zero rest mass of photon. Only moving photon has mass as follows from the Einstein formula $E = mc^2$. Mass of the moving photon is $m_\gamma = \hbar\omega/c^2$.

[1] Institute of Plasma Physics ASCR, Prague Asterix Laser System, PALS, Za Slovankou 3, 182 21 Prague 8, Czech Republic.

[2] To whom correspondence should be addressed at Laboratory of Plasma Physics, Department of Physical Electronics, Masaryk University, Kotlářská 2, 611 37 Brno, Czech Republic; e-mail: pamir@physics.muni.cz.

A nonzero photon mass would have several implications, such as a frequency-dependent speed of light and the existence of longitudinal electromagnetic waves. Photon with the nonzero rest mass is evidently in contradiction with special relativity. Arnold Sommerfeld (1954), who first considered superluminal velocities and theoretically discovered the Čerenkov effect, wrote no remark on the massive photons in his famous Optics.

If we suppose that the momentum of massive photon is $p = \hbar\omega/c$, then from the Einstein formula follows that the energy of massive photon is $E = \sqrt{\hbar^2\omega^2 + m^2c^4}$.

The corresponding Planck formula for the density $P(\omega)$ of the black body radiation is as follows:

$$P(\omega) = \left(\frac{\omega^2}{\pi^2 c'^3}\right) \frac{E}{e^{E/kT} - 1}; \quad E = \sqrt{\hbar^2\omega^2 + m^2c^4}, \tag{3}$$

where we used the frequency ω instead of the momentum of photon, because the frequency is used in experiment and not momentum of photon. The massless limit of the formula (3) is the original Planck law. Quantity E given by Eq. (3) is in harmony with the quantum definition of massive photon (Pardy, 2002). The quantity c' is the velocity of photons inside the black body and it must be involved in the number of electromagnetic modes inside the blackbody. We can put approximately $c' \approx c$. To our knowledge there is no experimental evidence that the modified Planck law is correct. It means that massive photons cannot be involved into the theory of the black body radiation. To our knowledge the precise measurement of the anomalous magnetic moment of electron and Lamb shift agree with QED formulas with zero photon rest mass. On the other hand, if photons are moving in electromagnetic field, then they have nonzero rest mass (Ritus, 1969). This mass is a complex quantity, while we will consider here only real quantity. It follows from the polarization operator in external fields. This operator substantially differs from the operator in the dielectric medium (Pardy, 1994), where the fundamental role plays the index of refraction. Polarization of vacuum can be determined also by the source theory methods (Dittrich, 1978; Schwinger, 1970; Schwinger *et al.*, 1976). The photon mass following from the vacuum polarization is not generated by the Higgs mechanism or by the Schwinger mechanism. This mass is of the dynamical origin corresponding to the radiative corrections. To our knowledge, the experiments with the black body radiation in magnetic or electric field was never performed.

The formal introduction of the rest mass of photon exist in quantum electrodynamics, where for instance the processes with soft photons are calculated. In these calculations, the photon mass is introduced in order to avoid the infrared divergences (Berestetzkii *et al.*, 1989).

We know that introducing the nonzero photon mass modifies Coulomb law (Pardy, 2002). Such modification is discussed in literature (Okun, 1981). It is evident that massive photons play crucial role in gravity. However, this problem was not discussed in the prestige articles (Okun, 2002).

On the other hand, the possibility that photon may be massive particle has been treated by many physicists. The discussion is also devoted to the existence of the mass of neutrino and its oscillations, which can form some analogue with the photons with the same importance. The established fact is that the massive electrodynamics is a perfectly consistent classical and quantum field theory (Feldmann and Mathews, 1963; Goldhaber and Nieto, 1971; Minkowski and Seiler, 1971). In all respect the quantum version has the same status as the standard QED. In this article we do not solve the radiative problems in sense of article by van Nieuwenhuizen (1973). Our goal is to determine the Volkov solution of the Dirac equation with massive photons. The resulting equation is the Riccati equation which cannot be solved in general. So, we derive only some approximative formulas.

In particle physics and quantum field theory (Ryder, 1985; de Wit and Smith, 1986; Commins and Bucksbaum, 1983), photon is defined as a massless particle with spin 1. Its spin is along or in opposite direction to its motion. The massive photon as a neutral massive particle is usually called vector boson. The equation for vector boson was derived in the unified theory of the electro-weak interactions. There are other well-known examples of massive spin-1 particles. For instance neutral ϱ-meson, φ-meson, and J/ψ particle, bosons $W^{\pm}$ and Z^0 in particle physics.

While massless photon is described by the Maxwell Lagrangian, the massive photon is described by the Proca Lagrangian from which the field equations follow. The massive electrodynamics can be considered as a generalization of massless electrodynamics. The well-known area where the massive photon or boson plays substantial role is the theory of superconductivity (Ryder, 1985), plasma physics (Anderson, 1963), waveguides, and so on. Of course the mass of photon is not the relativistic vacuum rest mass, but effective mass which is generated by the physical properties of medium, or by some mechanism such as the Higgs mechanism, Schwinger mechanism, and so on. In this sense, the physics of massive photon is meaningful, the generalized Volkov solution of the Dirac equation with massive photon field is physically meaningful too and it is worthwhile to investigate problems with the massive photons.

In order to be pedagogically clear, we derive in Section 2 the Volkov solution of the Dirac equation for massless photon field. In Section 3, we find the Riccati equation which involves mass of photon. Then we discuss the emission of massive photons by electron in the periodic and δ-form electromagnetic field. We derive generalized Compton formulas for interaction of the multiphotonic object with electron.

2. VOLKOV SOLUTION OF THE DIRAC EQUATION WITH MASSLESS PHOTONS

Let us remember the derivation of the Volkov (1935) solution of the Dirac equation in vacuum (we use here the method of derivation and metric convention of Berestetzkii *et al.* (1989)):

$$(\gamma(p - eA) - m)\Psi = 0. \tag{4}$$

where

$$A^\mu = A^\mu(\varphi); \quad \varphi = kx. \tag{5}$$

We suppose that the four-potential satisfies the Lorentz gauge condition

$$\partial_\mu A^\mu = k_\mu (A^\mu)' = (k_\mu A^\mu)' = 0, \tag{6}$$

where the prime denotes derivation with regard to φ. From the last equation follows

$$kA = \text{const} = 0, \tag{7}$$

because we can put the constant to zero. The tensor of electromagnetic field is

$$F_{\mu\nu} = k_\mu A'_\nu - k_\nu A'_\mu. \tag{8}$$

Instead of the first-order Dirac equation (4), we consider the second-order equation that we get by multiplication of the linear equation by operator $(\gamma(p - eA) + m)$ (Berestetzkii *et al.*, 1989). We get

$$\left[(p - eA)^2 - m^2 - \frac{i}{2} e F_{\mu\nu}\sigma^{\mu\nu}\right]\psi = 0. \tag{9}$$

Using $\partial_\mu(A^\mu\psi) = A^\mu\partial_\mu\psi$, which follows from Eq. (6), and $\partial_\mu\partial^\mu = \partial^2 = -p^2$, with $p_\mu = i(\partial/\partial x^\mu) = i\partial_\mu$, we get the second-order Dirac equation for the four potential of the plane wave:

$$[-\partial^2 - 2i(A\partial) + e^2A^2 - m^2 - ie(\gamma k)(\gamma A')]\psi = 0. \tag{10}$$

We look for the solution of the last equation in the form

$$\psi = e^{-ipx}\, F(\varphi). \tag{11}$$

After insertion of this equation into (10), we get with ($k^2 = 0$)

$$\partial^\mu F = k^\mu F', \quad \partial_\mu\partial^\mu F = k^2 F'' = 0, \tag{12}$$

the following equation for $F(\varphi)$:

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A')]F = 0. \tag{13}$$

The integral of the last equation is of the form

$$F = \exp\left\{-i\int_0^{kx}\left[\frac{e(pA)}{(kp)} - \frac{e^2}{2(kp)}A^2\right]d\varphi + \frac{e(\gamma k)(\gamma A)}{2(kp)}\right\}\frac{u}{\sqrt{2p_0}}, \tag{14}$$

where $u/\sqrt{2p_0}$ is the arbitrary constant bispinor.

All powers of $(\gamma k)(\gamma A)$ above the first are equal to zero, since

$$(\gamma k)(\gamma A)(\gamma k)(\gamma A) = -(\gamma k)(\gamma k)(\gamma A)(\gamma A) + 2(kA)(\gamma k)(\gamma A) = -k^2 A^2 = 0. \quad (15)$$

Then, we can write

$$\exp\left\{e\frac{(\gamma k)(\gamma A)}{2(kp)}\right\} = 1 + \frac{e(\gamma k)(\gamma A)}{2(kp)}. \quad (16)$$

So, the solution is of the form

$$\Psi_p = R\frac{u}{\sqrt{2p_0}}e^{iS} = \left[1 + \frac{e}{2kp}(\gamma k)(\gamma A)\right]\frac{u}{\sqrt{2p_0}}e^{iS}, \quad (17)$$

where u is an electron bispinor of the corresponding Dirac equation

$$(\gamma p - m)u = 0. \quad (18)$$

The mathematical object S is the classical Hamilton–Jacobi function, which was determined in the form

$$S = -px - \int_0^{kx} \frac{e}{kp}\left[(pA) - \frac{e}{2}(A)^2\right] d\varphi. \quad (19)$$

The current density is

$$j^\mu = \bar{\Psi}_p \gamma^\mu \Psi_p, \quad (20)$$

where $\bar{\Psi}$ is defined as the transposition of (17), or,

$$\bar{\Psi}_p = \frac{\bar{u}}{\sqrt{2p_0}}\left[1 + \frac{e}{2kp}(\gamma A)(\gamma k)\right] e^{-iS}. \quad (21)$$

After insertion of Ψ_p and $\bar{\Psi}_p$ into the current density, we have

$$j^\mu = \frac{1}{p_0}\left\{p^\mu - eA^\mu + k^\mu\left(\frac{e(pA)}{(kp)} - \frac{e^2A^2}{2(kp)}\right)\right\}, \quad (22)$$

which is in agreement with formula in the Meyer article (Meyer, 1971).

The so-called kinetic momentum corresponding to j^μ is as follows:

$$J^\mu = \Psi_p^*(p^\mu - eA^\mu)\Psi_p = \bar{\Psi}_p\gamma^0(p^\mu - eA^\mu)\Psi_p$$
$$= \left\{p^\mu - eA^\mu + k^\mu\left(\frac{e(pA)}{(kp)} - \frac{e^2A^2}{2(kp)}\right)\right\} + k^\mu\frac{ie}{8(kp)p_0}F_{\alpha\beta}(u^*\sigma^{\alpha\beta}u), \quad (23)$$

where

$$\sigma^{\alpha\beta} = \frac{1}{2}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha). \quad (24)$$

3. VOLKOV SOLUTION OF THE DIRAC EQUATION FOR MASSIVE PHOTONS

The original Volkov solution is based on the assumption that photon has zero rest mass, or, $k^2 = 0$. Our goal is to consider the solution of the Dirac equation in case that $k^2 = M^2$, where M is the rest mass of photon. We use here the metrical notation of Berestetzkii *et al.* (1989).

We apply the procedure of the preceding section for the case of the massive photon, and we write

$$\psi = e^{-ipx} F(\varphi), \tag{25}$$

where for F we get the following equation

$$M^2 F'' - 2i(kp)F' + G(\varphi)F = 0 \tag{26}$$

with

$$G(\varphi) = 2e(pA) - e^2 A^2 + ie(\gamma k)(\gamma A') \tag{27}$$

The Eq. (26) differs from the original Volkov equation (13) only by means of the massive term. However, the equation is substantially new, because of the second derivative of the function F. The solution of the last equation can be easily obtained in the approximative form in case that $M \to 0$. However, let us try to find the exact solution, which was not described, to our knowledge, in physical or mathematical journals.

In order to find such solution, we transcribe this equation in the form:

$$F'' + aF' + bF = 0, \tag{28}$$

where

$$a = -\frac{2i(kp)}{M^2}, \quad b(\varphi) = \frac{G(\varphi)}{M^2}. \tag{29}$$

Using the substitution

$$F = v(\varphi)e^{-\frac{1}{2}a\varphi}, \tag{30}$$

we get simple equation for $v(\varphi)$:

$$v'' + P(\varphi)v = 0, \tag{31}$$

where

$$P(\varphi) = -\frac{a^2}{4} + b. \tag{32}$$

Using the substitution

$$v(\varphi) = e^{\int_0^{\varphi} T(\varphi)\, d\varphi}, \tag{33}$$

we get from Eq. (31)

$$T' + T^2 + P(\varphi) = 0. \tag{34}$$

Equation (34) is so-called Riccati equation. The mass term is hidden in $P(\varphi)$. It is well know that there is no general form of solution of this equation. There is only some solution expressed in the elementary functions for some specific functions $P(\varphi)$. Nevertheless, there is interesting literature concerning the Riccati equation. For instance, Riccati equation is applied in the supersymmetric quantum mechanics (Cooper *et al.*, 1995), in variational calculus (Zelekin, 1998), nonlinear physics (Matveev and Salle, 1991), in renormalization group theory (Buchbinder *et al.*, 1992; Milton *et al.*, 2001) and in thermodynamics (Rosu and de la Crus, 2001).

With regard to circumstances, we are forced to find some approximative solution with form similar to the original Volkov solution. Let us show the derivaton of such approximative solution. We hope it will play the same role in quantum electrodynamics with the massive photon as in the case with the massless photon.

There are many approximative methods for solution of this problem. We choose the elementary method which was also applied to the Schrödinger equation and which is described for instance in the monograph of Mathews and Walker (1964).

The approximation consists at the application of the following inequalities:

$$|F''(\varphi)| \ll |F'(\varphi)|; \quad |F''(\varphi)| \ll |F(\varphi)|. \tag{35}$$

Then, we get the original Volkov solution with the difference that the existence of the nonzero photon mass will be involved only in the exponential expansion. Or, with $U = e(\gamma k)(\gamma A)/2(kp)$, we perform the expansion:

$$\begin{aligned} e^U &= \left\{1 + \frac{1}{1!}U + \frac{1}{2!}U^2 + \frac{1}{3!}U^3 + \cdots\right\} \\ &= \left\{1 + \frac{e}{2(kp)}(\gamma k)(\gamma A) + \frac{1}{2!}\left(\frac{e}{2(kp)}\right)^2(-M^2A^2)\right. \\ &\quad \left. + \frac{1}{3!}\left(\frac{e}{2(kp)}\right)^3(\gamma k)(\gamma A)(-M^2A^2) + \frac{1}{4!}\left(\frac{e}{2(kp)}\right)^4(M^4A^4) + \cdots\right\}, \end{aligned} \tag{36}$$

where we have used Eq. (15) in the modified form

$$\begin{aligned} (\gamma k)(\gamma A)(\gamma k)(\gamma A) &= -(\gamma k)(\gamma k)(\gamma A)(\gamma A) + 2(kA)(\gamma k)(\gamma A) \\ &= -k^2A^2 = -M^2A^2, \end{aligned} \tag{37}$$

with $k^2 = M^2$ for massive photons. We see that in this method of approximation the Massive solution involves the Volkov solution as the basic term and then the additional terms containing photon mass.

After performing some algebraic operations, we get the first approximation of the Volkov solution with the massive photon in the following form

$$\Psi_p = R(A, M^2)\frac{u}{\sqrt{2p_0}}e^{iS}$$
$$= \left[1 + \frac{e}{2kp}(\gamma k)(\gamma A) - \left(\frac{e}{2kp}\right)^2 M^2 A^2 + \cdots\right]\frac{u}{\sqrt{2p_0}}e^{iS}. \quad (38)$$

Now, we are prepared to solve some physical problems with the Volkov solution with massive photons.

4. EMISSION OF MASSIVE PHOTONS BY ELECTRON MOVING IN THE PERIODIC FIELD

Let us consider the monochromatic circularly polarized electromagnetic wave with the four potential

$$A = a_1 \cos\varphi + a_2 \sin\varphi; \quad a_3 = 0; \quad \varphi = kx \quad (39)$$

with $k^\mu = (\omega, \mathbf{k})$ being a wave four-vector and $k^2 = M^2$, the four-amplitudes a_1 and a_2 are the same and one another perpendicular, or

$$a_1^2 = a_2^2 = a^2; \quad a_1 a_2 = 0. \quad (40)$$

We shall also use the Lorentz gauge condition, which gives $a_1 k = a_2 k = 0$.

The wave function is then of the form:

$$\psi_p = \left\{1 + \frac{e}{2(kp)}\left[(\gamma k)(\gamma a_1)\cos\varphi + (\gamma k)(\gamma a_2)\sin\varphi - \frac{e}{2(kp)}a^2 M^2 + \cdots\right]\right\}$$
$$\times \frac{u(p)}{\sqrt{2q_0}}\exp\left\{-ie\frac{a_1 p}{(kp)}\sin\varphi + ie\frac{a_2 p}{(kp)}\cos\varphi - iqx\right\}, \quad (41)$$

where

$$q^\mu = p^\mu - e^2\frac{a^2}{2(kp)}(k^\mu) \quad (42)$$

is the time-averaged value of Eq. (23).

The corresponding matrix element is of the obligate form (Berestetzkii *et al.*, 1989).

After performing the appropriate mathematical operation we get the δ-function in the matrix element, from which the conservations laws follow in the form

$$sk + q = q' + k' \quad (43)$$

The interpretation of this formula is as follows: s massive photons with momentum k are absorbed by electron with momentum q and only one massive photon is emitted with the four-vector k', and the final momentum of electron is q'. So, we see that the Volkov solution gives the multiphoton processes, which are intensively studied in the modern physics (Delone and Krainov, 2000).

For the periodic wave it is

$$q^2 = q'^2 = m_*^2; \quad m_* = m\sqrt{1 + \frac{e^4a^4}{(2kp)^2}\frac{M^2}{m^2} - \frac{e^2}{m^2}a^2} \tag{44}$$

which can be interpreted as a mass shift of electron in the periodic field, or, the mass renormalization.

If we consider an electron at a rest ($\mathbf{q} = 0$, $q_0 = m_*$), then from the formula (42), (43), and (44) follows

$$(s^2 + 1)\frac{M^2}{2m_*}\frac{1}{\omega\omega'} + s\frac{1}{\omega'} - \frac{1}{\omega} = \frac{s}{m_*}(1 - \cos\Theta); \quad s = 1, 2, 3, \ldots, n. \tag{45}$$

The massless limit of the last formula is the well-known Compton formula (with $M = 0$)

$$\omega' = \frac{s\omega}{1 + \frac{s\omega}{m_*}(1 - \cos\theta)}, \tag{46}$$

where θ is an angle between $\mathbf{k}$ and $\mathbf{k}'$. So we see that frequencies ω' are harmonic frequencies of ω.

5. EMISSION OF MASSIVE PHOTONS BY ELECTRON MOVING IN THE IMPULSIVE FORCE

We use the δ-function form of the ultrashort laser pulse (Pardy, 2003)

$$A_\mu = a_\mu\eta(\varphi), \tag{47}$$

where $\eta(\varphi)$ is the Heaviside unit step function defined as follows: $\eta(\varphi) = 0$, $\varphi < 0$; and $\eta(\varphi) = 1$, $\varphi \geq 0$. Then, the function S and R in the Volkov solution Ψ_p are as follows (Pardy, 2003):

$$S = -px - \left[e\frac{ap}{kp} - \frac{e^2}{2kp}a^2\right]\varphi, \quad R = \left[1 + \frac{e}{2kp}(\gamma k)(\gamma a)\eta(\varphi) + \cdots\right]. \tag{48}$$

So, we get the matrix element in the form:

$$M = g\int d^4x\bar{\Psi}_{p'}O\Psi_p\frac{e^{ik'x}}{\sqrt{2\omega'}}, \tag{49}$$

where $O = \gamma e'^*$, $g = -ie^2$ in case of the electromagnetic interaction and

$$\bar{\Psi}_{p'} = \frac{\bar{u}}{\sqrt{2p'_0}}\bar{R}(p')\,e^{-iS(p')}. \tag{50}$$

In such a way, using above definitions, we write the matrix element in the form:

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0 2p_0}} \int d^4x\, \bar{R}(p') O R(p)\, e^{-iS(p')+iS(p)}\, e^{ik'x}. \qquad (51)$$

The quantity $\bar{R}(p')$ follows immediately from Eq: (48), namely

$$\bar{R}' = \overline{\left[1 + \frac{e}{2kp'}(\gamma k)(\gamma a)\eta(\varphi) + \cdots\right]} = \left[1 + \frac{e}{2kp'}(\gamma a)(\gamma k)\eta(\varphi) + \cdots\right]. \quad (52)$$

Using

$$-iS(p') + iS(p) = i(p' - p) + i(\alpha' - \alpha)\varphi, \qquad (53)$$

where

$$\alpha = \left(e\frac{ap}{kp} - \frac{e^2}{2}\frac{a^2}{kp}\right), \quad \alpha' = \left(e\frac{ap'}{kp'} - \frac{e^2}{2}\frac{a^2}{kp'}\right), \qquad (54)$$

we get

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0 2p_0}} \int d^4x \bar{u}(p')\bar{R}(p') O R(p) u(p)\, e^{i(p'-p)x}\, e^{i(\alpha'-\alpha)\varphi}\, e^{ik'x}. \quad (55)$$

We get after x-integration:

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0 2p_0}} \bar{u}(p') R(p') O R(p) u(p) \delta^{(4)}(kl + p - k' - p'). \qquad (56)$$

We see from the presence of the δ-function in Eq. (56) that during the process of the interaction of electron with the laser pulse the energy–momentum conservation law holds good:

$$lk + p = k' + p'; \quad l = \alpha - \alpha'. \qquad (57)$$

The last equation describes the so called multiphoton process, which can be also described using Feynman diagrams and which are studied in the different form intensively in the modern physics of multiphoton ionization of atoms (Delone and Krainov, 2000; Pardy, 2003).

If we introduce the angle Θ between $\mathbf{k}$ and $\mathbf{k}'$, then, with $|\mathbf{k}| = \omega$ and $|\mathbf{k}'| = \omega'$, we get from the squared equation (57) in the rest system of electron, where $p = (m_*, 0)$, the following equation $k = (\omega, \mathbf{k})$:

$$(l^2 + 1)\frac{M^2}{2m_*}\frac{1}{\omega\omega'} + l\frac{1}{\omega'} - \frac{1}{\omega} = \frac{l}{m_*}(1 - \cos\Theta); \quad l = \alpha - \alpha', \qquad (58)$$

which is modification of the original equation for the Compton process

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1 - \cos\Theta). \qquad (59)$$

We see that the substantial difference between single photon interaction and δ-pulse interaction is the factor $l = \alpha - \alpha'$.

We know that the last formula of the original Compton effect can be written in the form suitable for the experimental verification, namely:

$$\Delta\lambda = 4\pi \frac{\hbar}{mc} \sin^2 \frac{\Theta}{2}, \tag{60}$$

which was used by Compton for the verification of the quantum nature of light (Rohlf, 1994).

Let us remark, the equation $lk + p = k' + p'$ is the symbolic expression of the nonlinear Compton effect and it concerns only the situation where l photons are absorbed at a single point, and it does not describe the process where electron scatters twice, or more, as it traverses the laser focus. The nonlinear Compton process was experimentally confirmed (Bulla *et al.*, 1996).

6. DISCUSSION

The present article is continuation of the author discussion on laser interaction with electrons (Pardy, 1998, 2001), where the Compton model of laser acceleration was proposed and author article (Pardy, 2003), where the δ-form laser pulse was considered.

The δ-form laser pulses are the idealization of the experimental situation in laser physics. It was demonstrated theoretically that at present time, the zeptosecond and subzeptosecond laser pulses of duration 10^{-21} to 10^{-22} s can be realized by the petawat lasers (Kaplan and Shkolnikov, 2002). The generation of the ultrashort laser pulses is the keen interest in development of laser physics.

We have derived modified Compton formulas which involve multiphoton interaction of laser beam with electron. In case of the periodic field, the multiplicity is formed by the natural numbers and in case of the δ-pulse, by number $l = \alpha - \alpha'$. This effect can be interpreted in such a way that the photonic object with s or l photons interacts simultaneously with one electron. We do not think that the photonic object is consequence of the Bose–Einstein condensation of photons in laser beam. It behaves as photonic elementary object and probably it can be used in the experiments in particle physics.

The Volkov solution of the Dirac equation for electromagnetic potential with massive photons concerns not only the superconductive medium but also the electron–positron plasma, ionosphere medium, photons in waveguides, or massive photons generated hypothetically during inflation (Prokopec and Woodard, 2003).

In superconductivity, photon is a massive spin 1 particle as a consequence of a broken symmetry of the Landau-Ginzburg Lagrangian. The Meissner effect can be used as an experimental demonstration that photon in a superconductor is a massive particle. Kirzhnitz and Linde (1972) proposed a qualitative analysis wherein they indicated that, as in the Landau–Ginzburg theory of superconductivity, the Meissner effect can also be realized in the Weinberg model. It was shown that the Meissner effect is realizable in renormalizable gauge fields and also in the Weinberg model (Yildiz, 1977).

The bosons $W^{\pm}$ and Z^0 are massive and it means that the generalization of our approach to the situation in the standard model is evidently feasible. The vector mesons $\rho, \varphi, J/\psi$ are generated during the nuclear collisions and probably, the Volkov solution for these massive vector particles will play substantial role in the nuclear physics.

ACKNOWLEDGMENTS

I thank Professor Jam Jarica from Masaryk University, and Department of PALS for support in laser physics.

REFERENCES

Anderson, P. W. (1963). Plasmons, gauge invariance, and mass. *Physical Review* **130**(1), 439.

Berestetzkii, V. B., Lifshitz, E. M., and Pitaevskii, L. P. (1989). *Quantum Electrodynamics*, Nauka, Moscow (in Russian).

Buchbinder, I. L., Odintsov, S. D., and Shapiro, I. L. (1992). *Effective Action in Quantum Gravity*, IOP Publishing, Bristol, UK.

Bulla, C., McDonald, K. T., Prebys, E. J., *et al.* (1996). Observation of nonlinear effects in Compton scattering. *Physical Review Letter* **76**, 3116.

Commins, E. D. and Bucksbaum, P. H. (1983). *Weak Interactions of Leptons and Quarks*, Cambridge University Press, Cambridge.

Cooper, F., Khare, A., and Sukhatme, U. (1995). Supersymmetry and quantum mechanics. *Physics Reports* **251**, 267.

Delone, N. B. and Krainov, V. P. (2000). *Multiphoton Processes in Atoms*, 2nd edn., Springer-Verlag, Berlin.

de Wit B. and Smith, J. (1986). *Field Theory in Particle Physics, Vol. 1*, Elsevier, Amsterdam.

Dittrich, W. (1978). Source methods in quantum field theory. *Fortschritte der Physik* **26**, 289.

Feldman, G. and Mathews, P. T. (1963). Massive electrodynamics. *Physical Review* **130**, 1633.

Goldhaber, A. S. and Nieto, M. M. (1971). Terrestrial and extraterrestrial limits on the photon mass. *Review of Modern Physics* **43**, 277.

Kaplan, A. E. and Shkolnikov, P. L. (2002). Lasetron: A proposed source of powerful nuclear-time-scale electromagnetic bursts. *Physical Review Letter* **88**(7), 074801.

Kirzhnitz, A. D. and Linde, A. D. (1972). Macroscopic consequences of the Weinberg model. *Physics Letters B* **42**, 471.

Mathews, J. and Walker, R. L. (1964). *Mathematical Methods of Physics*, W. B. Benjamin, New York.

Matveev, V. B. and Salle, M. A. (1991). *Darboux Transformations and Solitons*, Springer, Berlin.

Meyer, J. W. (1971). Covariant classical motion of electron in a laser beam. *Physical Review D: Particles and Fields* **3**(2), 621.

Milton, K., Odintsov, S. D., and Zerbini, S. (2001). Bulk versus brane running couplings. e-print hep-th/0110051.

Minkowski, P. and Seiler, R. (1971). Massive vector meson in external fields. *Physical Review D: Particles and Fields* **4**, 359.

Okun, L. B. (1981). *Leptons and Quarks*, Nauka, Moscow (in Russian).

Okun, L. B. (2002). Photons, clocks, gravity and the concept of mass. *Nuclear Physics B (Proc. Suppl.)* **110**, 151.

Pardy, M. (1994). The Čerenkov effect with radiative corrections. *Physics Letters B* **325**, 517.

Pardy, M. (1998). The quantum field theory of laser acceleration. *Physics Letters A* **243**, 223.

Pardy, M. (2001). The quantum electrodynamics of laser acceleration. *Radiation Physics and Chemistry* **61**, 391.

Pardy, M. (2002). Čerenkov effect with massive photons. *International Journal of Theoretical Physics* **41**(5), 887.

Pardy, M. (2003). Electron in the ultrashort laser pulse. *International Journal of Theoretical Physics* **42**(1), 99.

Prokopec, T. and Woodard, D. (2003). Vacuum polarization and photon mass in inflation. e-print astro-ph/0303358; CERN-TH/2003-065.

Ritus, V. I. (1969). Radiative effects and their enhancement in intensive electromagnetic field. *Journal of Experimental and Theoretical Physics* **57**(6), 2176 (in Russian).

Rohlf, J. W. (1994). *Modern Physics from α to Z^0*, Wiley, New York.

Rosu, A. C. and de la Crus, Aceves (2001). One-parameter Darboux-transformed quantum action in thermodynamics. e-print quant-ph/0107043.

Ryder, L. H. (1985). *Quantum Field Theory*, Cambridge University Press, Cambridge.

Schwinger, J. (1970). *Particles, Sources and Fields, Vol. 1*, Addison-Wesley, Reading, MA.

Schwinger, J., Tsai, W. Y., and Erber, T. (1976). Classical and quantum theory of synergic synchrotron-Čerenkov radiation. *Annals of Physics (New York)* **96**, 303.

Sommerfeld, A. (1954). *Optics*, Academic Press, New York.

van Nieuwenhuizen, P. (1973). Radiation of massive gravitation. *Physical Review D: Particles and Fields* **7**(8), 2300.

Volkov, D. M. (1935). Über eine Klasse von Lösungen der Diracschen Gleichung. *Zeitschrift für Physik* **94**, 250.

Yildiz, A. (1977). Meissner effect in gauge fields. *Physical Review D: Particles and Fields* **16**(12), 3450.

Zelekin, M. I. (1998). *Homogenous Spaces and Riccati Equation in Variational Calculus*, Factorial, Moscow (in Russian).